Modes of Strongly Guiding Curved Dielectric Slabs

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Abstract:

It was recently concluded that bends of radii suitable for Integrated Optics applications are only realizable using strongly guiding curved dielectric guides. While curved dielectric slabs of weak optical guidance and small curvatures have been extensively studied in the literature, strongly guiding curved slabs have not gained enough examination. This paper discusses guided mode propagation in strongly guiding, strongly curved dielectric slabs. It is found that differences in propagation constant between modes of such slabs can be made one order of magnitude larger than those between straight slab modes by controlling slab curvature. The Beam Propagation Method is used to verify the analytically obtained results.

I. Introduction

It was recently concluded that bends of radii suitable for Integrated Optics applications are only realizable using strongly guiding curved dielectric guides [1]. Such guides of rectangular cross section can be used for low-loss directional change to facilitate optical beam manipulation in a small size chip. Using the effective refractive index method reduces the analysis of a curved rectangular dielectric guide to the case of a curved dielectric slab [2,3]. Curved dielectric slabs of weak optical guidance, and small curvatures have been extensively studied in the literature [4-7]. Due to excessive radiation losses, only strongly guiding slabs of large curvatures are of interest. These have not yet gained enough examination [1,8].

In this paper, we present a study of guided modes in strongly guiding, strongly curved dielectric slab guides. In part II, the nature of propagation modes in curved slabs, and the classification of guided modes into straight-slab- like and edge guided modes are briefly discussed. Part III contains the analysis of guided modes in the strongly guiding case. Interesting characteristics of edge guided modes that appear in strongly guiding slabs are presented. In part IV, an independent verification of the validity of the analysis of guided modes, of part III, is carried out using the Beam Propagation Method. The Beam Propagation algorithm used is described in the Appendix.

II. Guided Modes of the Curved Dielectric Slab

Azimuthally-propagating modes of the curved dielectric slab have either their electric or magnetic fields parallel to the slab boundaries. For an TE mode, this electric field is given by :

$$\Phi = C_\nu(r)\, e^{-j\nu\phi} \tag{1}$$

where r, and ϕ are the radial and azimuthal coordinates, respectively, j is the square root of (-1), and an $e^{j\omega t}$ factor is suppressed. ν, the azimuthal propagation constant, is the mode eigen value which is generally complex, and $C_\nu(r)$ is the corresponding eigen function, giving the radial dependence of the modal field. ν can be expressed as :

$$\nu = \nu_r - j\,\nu_i \tag{2.a}$$

$$\nu_i < \nu_r \tag{2.b}$$

where a non-severely attenuated mode is assumed.

Substituting (1) into the Wave equation gives $C_\nu(r)$ in terms of Bessel functions of the order ν.

Eliminating arbitrary constants using electric and magnetic field continuity conditions results in a very complicated eigen value equation [2,3,5]. Nevertheless, a determination of the range of values of the real part, ν_r, for guided mode solutions is possible using geometrical optics.

Each guided mode can be considered as a group of rays, or local plane waves having propagation and attenuation factors given by :

$$\nu\,\phi = (\beta - j\,\alpha)\, r\,\phi \tag{3.a}$$

$$\beta = \frac{\nu_r}{r}\,, \qquad \alpha = \frac{\nu_i}{r} \tag{3.b}$$

Due to curvature, both factors are inversely proportional to the radial coordinate. All guided rays in the curved slab suffer total internal reflection at the outer slab boundary. Reflection of optical rays at the outer slab boundary is a frustrated total internal reflection which gives rise to curvature radiation losses [9]. Some guided rays suffer total internal reflection at both slab boundaries. Meanwhile, other rays turn to propagate in the azimuthal direction before reaching the inner boundary. Fig.1 illustrates both categories of guided rays.

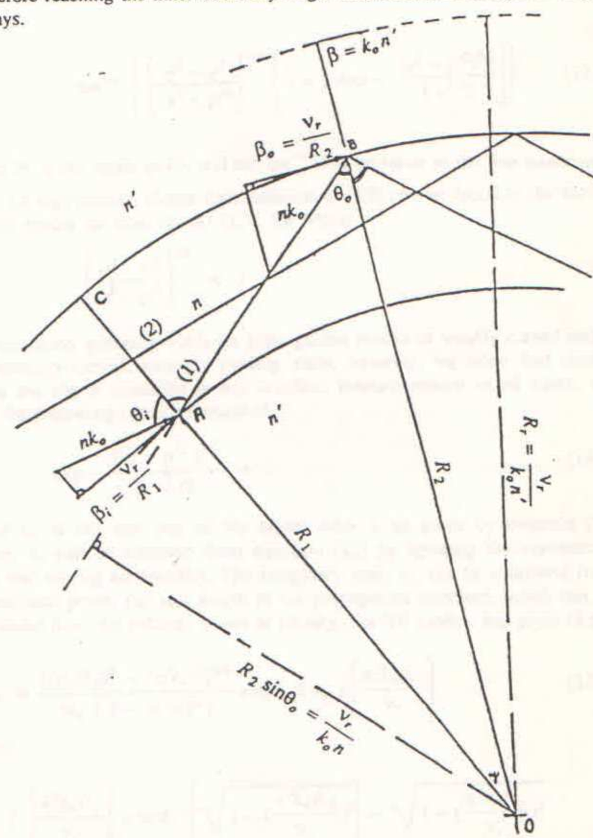

Fig.1 Ray optics representation of guidance in a curved slab waveguide. Rays (1), and (2) are slab-, and edge-guided, respectively.

Correspondingly, two categories of guided modes of the curved slab exist. For a curved slab of inner and outer radii of curvature of R_1, and R_2, and core and cladding refractive indices of n, and n', respectively, both categories have values of ν_r that satisfy [3,4]:

$$n\,k_o\,R_2 \geq \nu_r \geq n'\,k_o\,R_2 \tag{4}$$

where k_o is the free space wave number. The first category is straight-slab-like modes, for which :

$$n\,k_o\,R_1 \geq \nu_r \geq n'\,k_o\,R_2 \tag{5}$$

The second category is edge-guided modes, which have :

$$n\,k_o\,R_2 \geq \nu_r \geq n\;\;k_o\,R_1 \tag{6}$$

In a strongly guiding slab with $n\,R_1 \leq n'\,R_2$, only edge guided modes exist.

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Instead of formulating modal solutions in terms of Bessel functions, a conformal mapping can be used to reduce the curved slab structure to an equivalent nonhomogeneous straight slab [10]. If R_c is the radius of curvature at the centre of a curved slab of refractive index $n(r)$, the transformation :

$$w = R_c \ \ln \frac{Z}{R_c} \tag{7}$$

where $w = u + j\, v$, $Z = r\, e^{j\phi}$

reduces the curved slab to an equivalent straight one with refractive index :

$$n(u) = n(r)\ e^{u/R_c} \tag{8}$$

A mode of the curved slab with eigen value ν corresponds to a mode of the equivalent straight slab with eigen value :

$$\beta_c - j\ \alpha_c = \frac{\nu}{R_c} \tag{9}$$

The modal field at the coordinate u of the straight slab, and hence at the corresponding radius r of the curved slab, is either evanescent or oscillating depending on whether the propagation constant, β_c is greater or less than $k_o n(u)$, respectively. Fig.2 illustrates the transformed slab with the two categories of guided modes.

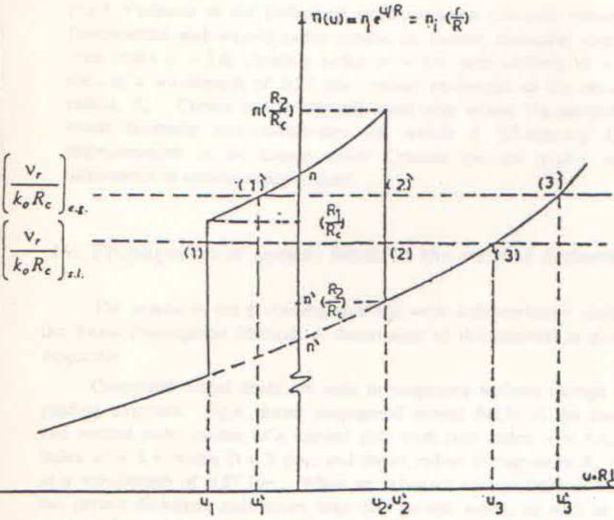

Fig.2 Refractive index profile of the equivalent straight slab, and modal propagation constants for straight-slab-like, and edge-guided modes.

III. Analysis of guided modes in strongly guiding slabs

1) Straight-slab-like modes

As can be seen from Fig.2, straight-slab-like modes have optical fields with oscillatory radial profiles within the slab core, while evanescent in the claddings. Both in strongly and weakly guiding slabs, these modes can be analyzed using a WKB approximation [3,5]. A comparison of published treatments of straight-slab-like modes in weakly guiding slabs and a direct application of a regular WKB approximation to these modes in strongly guiding slabs can be found in Ref.[3].

2) Edge guided modes

2.a. The Whispering Gallery approximation : Edge guided modes have field profiles that are evanescent with decreasing radius at some point in the slab core before reaching the inner boundary and hence the presence of the this boundary has only a little effect on their properties. This effect can be ignored in an approximate treatment of the modes and hence only the outer boundary is taken into consideration. This approximation is known as the Whispering Gallery approximation [11]. For Whispering Gallery modes, $C_\nu(r)$ can be written as :

$$C_\nu(r) = \begin{cases} A\ J_\nu(k_o n r) & : r \le R_2 \\ A\ \dfrac{J_\nu(k_o n\, R_2)}{H_\nu^{(2)}(k_o n' R_2)}\ H_\nu^{(2)}(k_o n' r) & : r \ge R_2 \end{cases} \tag{10}$$

where J_ν, and $H_\nu^{(2)}$ are Bessel function of the first kind, and Hankel function of the second kind, respectively, of the same order ν. The characteristic equation for TE modes simplifies to :

$$\frac{n'}{n}\ \frac{H_\nu^{(2)\prime}(\rho')}{H_\nu^{(2)}(\rho')} = \frac{J_\nu'(\rho)}{J_\nu(\rho)} \tag{11}$$

where ρ , and ρ' are equal to nk_oR_2, and $n'k_oR_2$, respectively, and the derivatives are taken with respect to the arguments. For the values of J_ν and $H_\nu^{(2)}$ involved in equation (11) we use the Debye Asymptotic expansions of Bessel functions [12], for which the condition given by equation (4) for guided modes holds. Substituting these asymptotic expansions into equation (11) gives the following characteristic equation [2,3]:

$$\frac{(\rho^2 - \nu^2)^{3/2}}{3\,\nu^2} \quad = \quad (M - \frac{1}{4})\pi$$

$$- \tan^{-1} \left\{ \left[\frac{\rho^2 - \nu^2}{\nu^2 - \rho'^2} \right]^{1/2} \left[1 + j\ \exp - \frac{(\nu^2 - \rho'^2)^{3/2}}{3\,\nu^2/2} \right] \right\} \tag{12}$$

where M is the mode order, and the $\tan^{-1}$ is to be taken in the first quadrant.

An approximate closed-form solution to (12) can be found in the literature for modes far from cut off [2,7], for which :

$$\left[\frac{\rho^2 - \nu^2}{\rho^2 - \rho'^2} \right]^{1/2} < 1 \tag{13}$$

This condition generally holds for edge guided modes of weakly curved slabs. For strongly curved, strongly guiding slabs, however, we often find modes where the above condition is not satisfied. Instead, almost in all cases, we have the following condition satisfied :

$$\exp - \frac{(\nu_r^2 - \rho'^2)^{3/2}}{3\,\nu_r^2/2} < 1 \tag{14}$$

where ν_r is the real part of the eigen value ν as given by equation (2). Hence, ν_r can be obtained from equation (12) by ignoring the exponential term and solving numerically. The imaginary part, ν_i, can be evaluated from the radiated power per unit length in the propagation direction, which can be calculated from the radiated power at infinity. For TE modes, this gives [3,6]:

$$\nu_i = \frac{[(\nu_r/R_2)^2 - (n'k_o)^2]^{1/2}}{nk_o\ [\,1 - (n'/n)^2\,]}\ \exp - 2\,\nu_r\, f\!\left[\frac{n'k_oR_2}{\nu_r}\right] \tag{15}$$

where

$$f\left[\frac{n'k_oR_2}{\nu_r}\right] = \tanh^{-1}\left[\sqrt{1 - (\frac{n'k_oR_2}{\nu_r})^2}\right] - \sqrt{1 - (\frac{n'k_oR_2}{\nu_r})^2}$$

2.b. Differences in modal propagation constants : After solving for ν of edge guided modes by the method outlined above, modal propagation factors at the centre of the curved slab can be calculated. Comparing differences in these propagation constants with those between modes of a similar straight slab, it is found that in strongly guiding slabs, differences in modal propagation constants can be controlled by slab curvature over almost an order of magnitude. Fig.3 shows variation of the difference in propagation constant between the fundamental and second order modes of curved dielectric slabs with core and cladding refractive indices 3.6, and 3.4, respectively, and different slab widthes.

The large differences in modal propagation constant between modes of a strongly guiding, strongly curved dielectric slab is justified since different modes of such a slab effectively propagate in regions of different equivalent refractive indices [3].

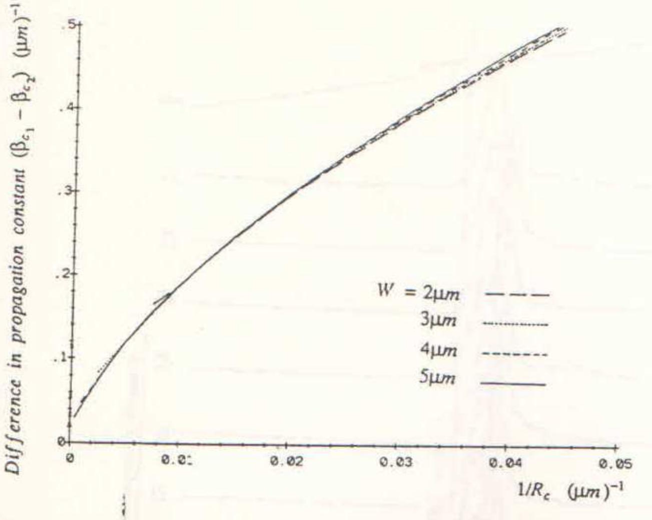

Fig.3 Variation of the difference in propagation constant between the fundamental and second order modes of curved dielectric slabs with core index $n = 3.6$, cladding index $n' = 3.4$, and widthes W = 2 - 5 μm, at a wavelength of 0.87 μm, versus reciprocal of the mean slab radius, R_c . Curves end at the left hand side where the second order mode becomes straight-slab-like for which a Whispering Gallery approximation is no longer valid. Crosses on the y-axis indicate differences in similar straight slabs.

IV. Propagation of optical fields in the curved dielectric slab

The results of the preceding analysis were independently verified using the Beam Propagation Method. A description of this method is given in the Appendix.

Computed modal fields are seen to propagate without change along the guiding structure. Fig.4 shows propagated modal fields of the fundamental and second order modes of a curved slab with core index $n = 3.6$, cladding index $n' = 3.4$, width D = 3 μm, and mean radius of curvature $R_c = 25$ μm, at a wavelength of 0.87 μm. When an arbitrary optical field is imposed on the curved dielectric slab, more than one guided mode, as well as radiation modes of the slab are excited. After sufficient propagation distance, radiation ceases and mode mixing clearly appears. Fig.5 gives the propagated field in a curved slab with the same parameters as above when excited by the lowest order mode of a straight similar slab. In (b) of this figure, the field is monitored every $\frac{L_b}{4}$ where L_b is the beating length between the two edge guided modes of the slab as calculated by the methods of section III.

V.Conclusion

Guided mode propagation in strongly guiding, strongly curved dielectric slabs was investigated. It is found that differences in propagation constant between modes of such slabs can be controlled by slab curvature over almost an order of magnitude. An independent verification of the analytically obtained results was carried out using the Beam Propagation Method. The analysis presented here completes the comprehensive study of curved dielectric slab modes. The BPM algorithm used can also be applied in the study of coupling, and field evolution in strongly guiding Integrated Optics bends.

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Appendix

The Beam Propagation Method provides a numerical solution to the scalar Wave equation using spectral transforms. The scalar Helmholtz equation can be written as :

$$\frac{\partial^2 E}{\partial^2 z} + \nabla_t^2 E = -k_o^2 \,(n + \delta n)^2 E \qquad \text{(A.1)}$$

where z is the direction of wave propagation, ∇_t^2 is the Laplacian in the transverse coordinates, k_o is the free space wave number, n is some constant index, and δn is the deviation of the medium refractive index from n. Neglecting $\frac{\partial(\delta n)}{\partial z}$, and the non-commutativity of ∇_t^2 and δn, and assuming that $\delta n \ll n$, equation (A.1) is satisfied if [13]:

$$\frac{\partial E}{\partial z} = -j\left[\frac{\nabla_t^2}{(\nabla_t^2 + k_o^2 n^2)^{1/2} + k_o n} + k_o n + k_o \delta n\right] E \qquad \text{(A.2)}$$

Solution to (A.2) over very small propagation distances takes the form [14]:

$$E(x,y,z+\Delta z) = exp(-jnk_o \Delta z)\, \exp\left[-j\frac{\Delta z}{2}\, \frac{\nabla_t^2}{(\nabla_t^2 + k_o^2 n^2)^{1/2} + k_o n}\right] \cdot$$

$$\exp\left[-jk_o \delta n\, \Delta z\right]\, \exp\left[-j\frac{\Delta z}{2}\, \frac{\nabla_t^2}{(\nabla_t^2 + k_o^2 n^2)^{1/2} + k_o n}\right] E(x,y,z) \qquad \text{(A.3)}$$

This is equivalent to propagating the field a distance $\frac{\Delta z}{2}$ in a homogeneous medium of index n, through which the field is operated upon by a propagator operator. Then correcting for the phase change resulting from the refractive index difference δn over the same distance, which is done by operating a phase correction operator on the field as calculated after $\frac{\Delta z}{2}$. This process is then repeated after each step of $D_z = \frac{\Delta z}{2}$.

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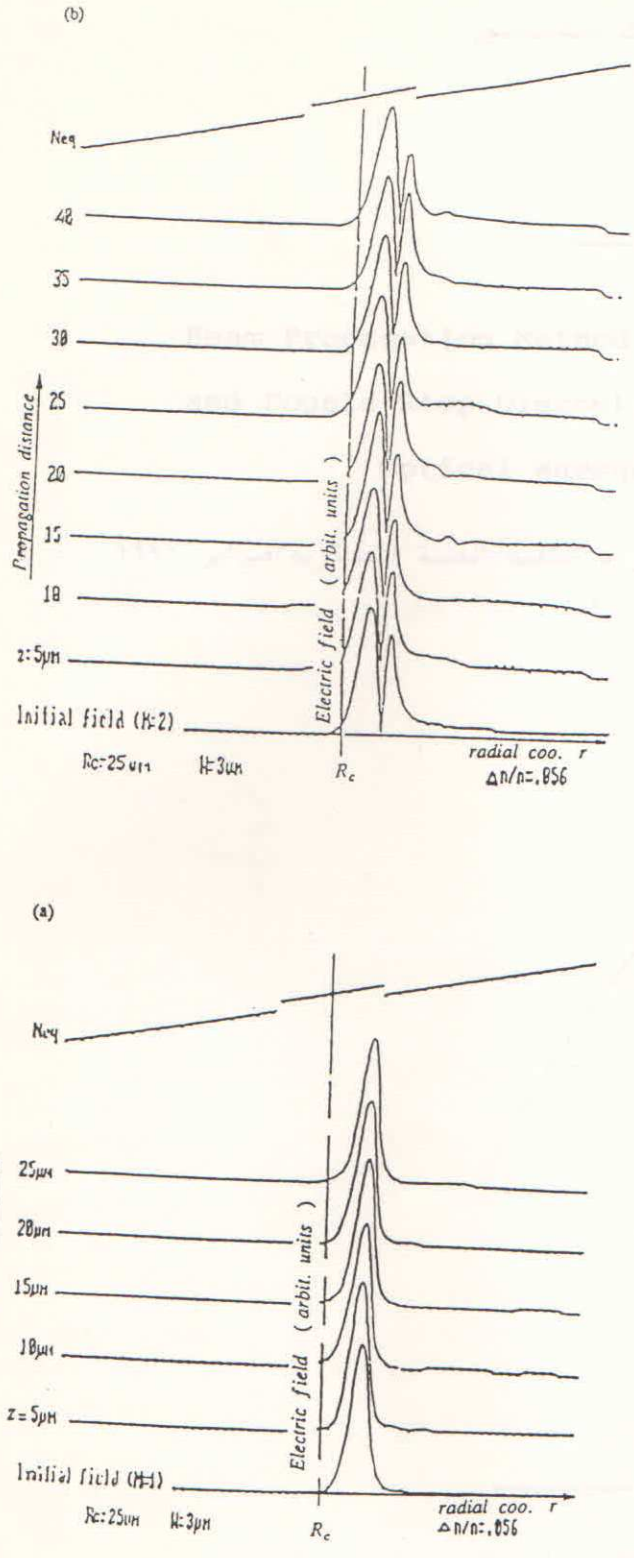

Fig.4 Propagated modal fields of the (a) fundamental, and (b) second order modes of a curved slab with the same parameters as those of Fig.3 with a width of 3 μm, and mean radius of curvature $R_c \doteq 25$ μm.

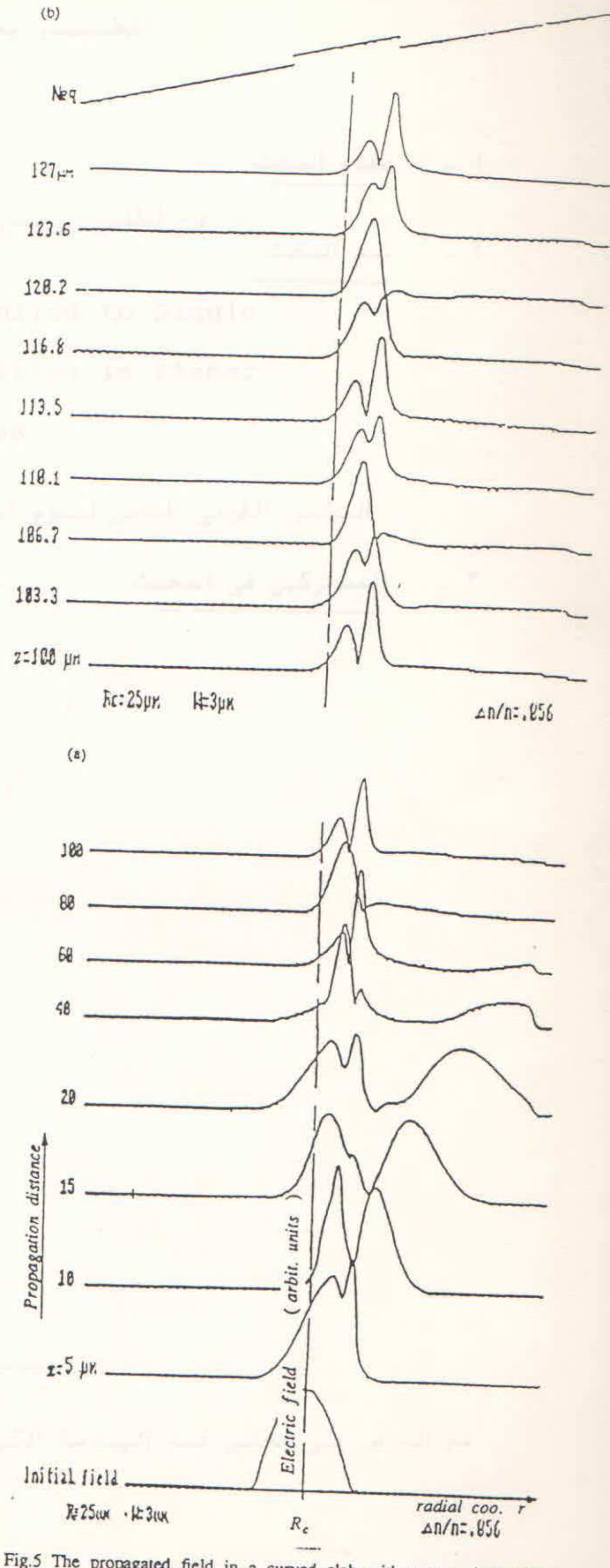

Fig.5 The propagated field in a curved slab with core and cladding refractive indices of 3.6, and 3.4, a width of 3 μm, and a bending radius of 25 μm when excited by the fundamental mode of a straight similar slab at a wavelength of 0.87 μm. In (b), the field is monitored every $\frac{L_b}{4}$ where L_b is the beating length between the two guided modes of the slab.